REPORT DOCUMENTATION PAGE

Form Approved
OMB NO. 0704-0188

1. AGENCY USE ONLY (Leave Blank	2. REPORT DATE 2/26/99	3. REPORT TYPE AND DATES COVERED Final

4. TITLE AND SUBTITLE PDE, Differential Geometric, Wavelet and Algebraic Methods in Nonlinear Filtering	5. FUNDING NUMBERS DAAH04-95-1-0530
6. AUTHOR(S) Stephen S.T. Yau	
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) Board of Trustees University of Illinois 809 S. Marshfield Ave Chicago IL 60612	8. PERFORMING ORGANIZATION REPORT NUMBER
9. SPONSORING / MONITORING AGENCY NAME(S) AND ADDRESS(ES) U. S. Army Research Office P.O. Box 12211 Research Triangle Park, NC 27709-2211	10. SPONSORING / MONITORING AGENCY REPORT NUMBER ARO 33422.8-MA

11. SUPPLEMENTARY NOTES

The views, opinions and/or findings contained in this report are those of the author(s) and should not be construed as an official Department of the Army position, policy or decision, unless so designated by other documentation.

12 a. DISTRIBUTION / AVAILABILITY STATEMENT	12 b. DISTRIBUTION CODE
Approved for public release; distribution unlimited.	

13. ABSTRACT (Maximum 200 words

We have found the best solution to Duncan-Mortensen-Zakai (DMZ) equation for Yau filtering system, which is the most general filtering system and includes both linear filtering and exact filtering systems. We show that this equation can be solved explicitly with an arbitrary initial condition by solving a system of ordinary differential equations and a Kolmogorov type equation. Let n be the dimension of the state space. We show that we need only n sufficient statistics in order to solve DMZ equation.

In the other direction, we prove that if the estimation algebra is finite dimensional of maximal rank, then the Wang's matrix has constant structures. this theorem plays a fundamental role in the classification of finite dimensional estimation algebra of maximal rank In fact we have classified all these Lie algebras for statespace dimension less than or equal to 6.

14. SUBJECT TERMS	15. NUMBER OF PAGES
Duncan-Mortensen-Zakai equation, Yau filtering system, estimation algebra	16. PRICE CODE

17. SECURITY CLASSIFICATION OR REPORT	18. SECURITY CLASSIFICATION ON THIS PAGE	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT
UNCLASSIFIED	UNCLASSIFIED	UNCLASSIFIED	UL

NSN 7540-01-280-5500

Standard Form 298 (Rev.2-89
Prescribed by ANSI Std. 239-18
298-102

PDE, DIFFERENTIAL GEOMETRIC, WAVELET AND ALGEBRAIC METHODS IN NONLINEAR FILTERING

Final Report

STEPHEN S.-T. YAU

March 23, 1999

U.S. Army Research Office

Army Grant DAAH-04-1-0530

Department of Mathematics, Statistics,
and Computer Sciences (M/C 249)
University of Illinois at Chicago
851 South Morgan Street
Chicago. Illinois 60607-7045

"Best Available Copy"

Typeset by $\mathcal{A}\mathcal{M}\mathcal{S}$-TEX

Table of Content

A. Statement of problem studied.

The filtering problem considered here is based on the following signal observation model:

$$\begin{cases} dx(t) = f\big(x(t)\big)dt + g\big(x(t)\big)dv(t) & x(0) = x_0 \\ dy(t) = h\big(x(t)\big)dt + dw(t) & y(0) = 0 \end{cases} \tag{1}$$

in which x, v, y and w are, respectively, $\mathbf{R}^n$, $\mathbf{R}^p$, $\mathbf{R}^m$ and $\mathbf{R}^m$ valued processes, and v and w have components which are independent, standard Brownian process. We further assume that $n = p$, f, h are C^∞ smooth, and that g is an orthogonal matrix. We will refer to $x(t)$ as the state of the system at time t and $y(t)$ as the observation at time t.

Let $\rho(t,x)$ denote the conditional probability density of the state given the observation $\{y(s); 0 \le s \le t\}$. It is well-known that $\rho(t,x)$ is given by normalizing a function, $\sigma(t,x)$, which satisfies the following Duncan-Mortensen-Zakai equation:

$$d\sigma(t,x) = L_0\sigma(t,x)dt + \sum_{i=1}^{m} L_i\sigma(t,x)dy(t), \sigma(0.x) = \sigma_0 \tag{2}$$

where

$$L_0 = \frac{1}{2}\sum_{i=1}^{n}\frac{\partial^2}{\partial x_i^2} - \sum_{i=1}^{n} f_i\frac{\partial}{\partial x_i} - \sum_{i=1}^{n}\frac{\partial f_i}{\partial x_i} - \frac{1}{2}\sum_{i=1}^{m} h_i^2$$

and for $i = 1, \dots, m$, L_i is the zero degree differential operator of multiplication by h_i. σ_0 is the probability density of the initial point, x_0.

Equation (2) is a stochastic partial differential equation. In real applications, we are interested in constructing robust state estimators from observed sample paths with some property of robustness. Davis studied this problem and proposed some robust algorithms. In our case, his basic idea reduces to refining a new unnormalized density

$$u(t,x) = \exp\left(-\sum_{i=1}^{m} h_i(x)y_i(t)\right)\sigma(t.x).$$

It is easy to show that $u(t,x)$ satisfies the following time varying partial differential equation

$$\begin{aligned} \frac{du}{dt}(t,x) &= L_0u(t,x) + \sum_{i=1}^{m} y_i(t)[L_0, L_i]u(t.x) \\ &\quad + \frac{1}{2}\sum_{i,j=1}^{m} y_i(t)y_j(t)\big[[L_0, L_i], L_j\big]u(t,x) \\ u(0,x) &= \sigma_0 \end{aligned} \tag{3}$$

where $[L_0, L_i]$ denotes the Lie bracket of L_0 and L_i.

Definition. The estimation algebra E of a filtering problem (1), is defined to be the Lie algebra generated by $\{L_0, L_1, \ldots, L_m\}$, or $E = \langle L_0. L_1, \ldots, L_m\rangle_{L.A.}$

If in addition there exists a potential function ψ such that $f_i = \frac{\partial \psi}{\partial x_i}$ for all $1 \leq i \leq n$, then the estimation algebra is called exact.

The estimation algebra is said to be with maximal rank if $x_i + c_i$ is in E for all $1 \leq i \leq n$ where c_i is a constant.

The problem is to solve explicitly equation (3) in real-time. In particular, we want to construct all possible finite dimensional filters via direct method or Wei-Norman approach. This includes solving the Brockett problem on classification of finite dimensional estimation algebras.

B. Summary of the most important results.

(I) In [Mi 1], Mitter pointed out that the innovation approach to nonlinear filtering theory is not, in general, explicitly computable. It was first proposed by Brockett and Clark [Br-Cl], Brockett [Br], and Mitter [Mi 1] to use estimation algebras to construct finite-dimensional filters. The idea is to use the Lie algebraic method to solve the Duncan-Mortensen-Zakai (DMZ) equation, which is a stochastic partial differential equation. By working on the robust form of the DMZ equation, we can reduce the complexity of the problem to that of solving a time-variant partial differential equation.

In the past decade, the Lie algebraic viewpoint has been remarkably successful, and recent works [T-W-Y], [D-T-W-Y], [Ya], [Ch-Ya], [Ch-Ya1], [Ch-Ya2], [C-L-Y1], [C-L-Y2], [Hu-Ya2], [Hu-Ya3], [Wo-Ya] have given us a deeper understanding of the DMZ equation, which was essential for progress in nonlinear filtering as well as in stochastic control. In fact it was Yau (cf. [Ya1], [Ya2]) who first used Lie algebraic method to discover the most general class of finite dimensional filters which include linear filters and exact filters as special cases. In [Ch], this general class of finite dimensional filters are called Yau filters.

In spite of the success of the Lie algebra method, it is extremely desirable to treat the DMZ equation by a direct method.

Despite its usefulness, the Kalman-Bucy filter is not perfect. One of its weaknesses is that it needs a Gaussian assumption on the initial data. The situation is more complex when the statistics of the initial condition are modeled by an arbitrary distribution. In the case where the linear filtering system (i.e., f, g, and h are linear functions in (1) is completely reachable and completely observable, Hazewinkel observed on p. 115 of [Ha] that the estimation algebra E (i.e., a Lie algebra generated by differential operators L_0, $h_1(x), \ldots, h_m(x)$) is the $2n+2$ dimensional Lie algebra with basis L_0, $\frac{\partial}{\partial x_1}, \ldots, \frac{\partial}{\partial x_n}$, $x_1, \ldots, x_n$, 1. Even in this case, the Wei-Norman approach used to find the solution of (3) is more complicated than the procedure in Theorem 1 below because not only must one solve a finite system of ordinary differential equation and a Kolmogorov equation, but also one has to integrate n partial differential equations corresponding to the operators $\frac{\partial}{\partial x_1}, \ldots, \frac{\partial}{\partial x_n}$. More important, if the linear system is not completely reachable or completely observable, then the basis of the estimation algebra is not explicitly known (although it can be computed). As a result, there is an additional disadvantage of the Wei-Norman approach, namely, one cannot write down the finite system of ordinary differential equation explicitly in the non-maximal rank case. The novelty of theorem 1 is

that our finite system of ordinary differential equations is explicitly written down and our procedure to get the solution of (3) is simpler than the Lie algebra approach. Most importantly, our theorem works for Yau filtering system which includes linear filtering system and exact filtering system as special cases.

Theorem 1. Consider the Yau filtering system (1) with arbitrary initial condition, i.e.,

(Y1) $f_i(x) = \ell_i(x) + \frac{\partial F}{\partial x_i}(x)$, $1 \le i \le n$, where $\ell_i = \sum_{j=1}^{n} d_{ij}x_j + d_i$ for $1 \le i \le n$ and F is a C^∞ function.

(Y2) $h_i(x) = \sum_{j=1}^{n} c_{ij}x_j + c_i$, $1 \le i \le m$.

(Y3) $\eta(x) := \sum_{i=1}^{n} f_i^2(x) + \sum_{i=1}^{n} \frac{\partial f_i}{\partial x_i} + \sum_{i=1}^{m} h_i^2(x) = \sum_{i,j=1}^{n} \eta_{ij}x_ix_j + \sum_{i=1}^{n} \eta_i x_i + \eta_0$, where η_{ij}, η_i and η_0 are constants.

Choose a C^∞ function $G(x)$ such that

$$\Delta G(x) + |\nabla G|^2(x) + 2\sum_{i=1}^{n} \ell_i(x)\frac{\partial G}{\partial x_i}(x) = \eta(x) - \sum_{i=1}^{n} \ell_i^2(x) - \sum_{i=1}^{n} \frac{\partial \ell_i}{\partial x_i}(x).$$

Then the solution $u(t,x)$ for the Duncan-Mortensen-Zakai equation (3) is reduced to the solution $\widetilde{u}(t,x)$ for the Kolmogorov equation

$$\frac{\partial \widetilde{u}}{\partial t}(t,x) = \frac{1}{2}\Delta\widetilde{u}(t,x) - \sum_{i=1}^{n}\left(\ell_i(x) + \frac{\partial G}{\partial x_i}(x)\right)\frac{\partial \widetilde{u}}{\partial x_i}(t,x) - \sum_{i=1}^{n}\left(\frac{\partial \ell_i}{\partial x_i}(x) + \frac{\partial^2 G}{\partial x_i^2}(x)\right)\widetilde{u}(t,x)$$

where

$$\widetilde{u}(t,x) = e^{c(t)+G(x)+\sum_{i=1}^{n} a_i(t)x_i - F(x+b(t))} u(t, x+b(x))$$

and $a_i(t)$, $b_i(t)$, and $c(t)$ satisfy the following system of ODEs: For $1 \le i \le n$

$$b_i'(t) - a_i(t) - \sum_{j=1}^{n} d_{ij}b_j(t) + \sum_{j=1}^{m} c_{ji}y_j(t) = 0 \tag{4}$$

$$a_i'(t) + \sum_{j=1}^{n} d_{ji}b_j'(t) - \frac{1}{2}\sum_{j=1}^{n}(\eta_{ij}+\eta_{ji})b_j(t) = 0 \tag{5}$$

$$\begin{aligned} c'(t) &+ \frac{1}{2}\sum_{i=1}^{n}(b_i'(t))^2 - \sum_{i=1}^{n} a_i(t)b_i'(t) - \frac{1}{2}\sum_{k,j=1}^{n} \eta_{ij}b_i(t)b_j(t) \\ &- \frac{1}{2}\sum_{i=1}^{n} \eta_i b_i(t) + \sum_{i=1}^{n} d_i b_i'(t) = 0. \end{aligned} \tag{6}$$

Theorem 2. Consider the Yau filtering system (1) with arbitrary initial condition satisfying (Y1), (Y2) and (Y3) in Theorem 1. Then the solution $u(t,x)$ for the Duncan-Mortensen-Zakai equation (3) is reduced to the solution $\widetilde{u}(t,x)$ for the Kolmogorov equation

$$\begin{aligned}\frac{\partial \widetilde{u}}{\partial t}(t,x) &= \frac{1}{2}\Delta\widetilde{u}(t,x) - \sum_{i=1}^{n} \ell_i(x)\frac{\partial \widetilde{u}}{\partial x_i}(t,x)\\ &+ \frac{1}{2}\left(\sum_{i=1}^{n} \ell_i^2(x) - \sum_{i=1}^{n}\frac{\partial \ell_i}{\partial x_i}(x) - \eta(x)\right)\widetilde{u}(t,x)\end{aligned}$$

where

$$\widetilde{u}(t,x) = e^{c(t)+\sum_{i=1}^{n} a_i(t)t_i - F(x+b(t))}u(t,x+b(t))$$

and $a_i(t)$, $b_i(t)$ and $c(t)$ satisfy ODEs (4), (5) and (6).

Theorem 3. Consider the Yau filtering system (1) with arbitrary initial condition satisfying (Y1), (Y2) and (Y3) in Theorem 1. Then the solution $u(t,x)$ for the Duncan-Mortensen-Zakai equation (3) is reduced to the solution $\widetilde{u}(t,x)$ for the Kolmogorov equation

$$\begin{aligned}\frac{\partial \widetilde{u}}{\partial t}(t,x) &= \frac{1}{2}\Delta\widetilde{u}(t,x) - \sum_{i=1}^{n} f_i(x)\frac{\partial \widetilde{u}}{\partial x_i}(t,x)\\ &+ \frac{1}{2}\left(\sum_{i=1}^{n} f_i^2(x) - \sum_{i=1}^{n}\frac{\partial f_i}{\partial x_i}(x) - \eta(x)\right)\widetilde{u}(t,x)\end{aligned}$$

where

$$\widetilde{u}(t,x) = e^{c(t)+\sum_{i=1}^{n} a_i(t)x_i + F(x) - F(x+b(t))}u(t,x+b(t))$$

and $a_i(t)$, $b_i(t)$ and $c(t)$ satisfy ODEs (4), (5) and (6).

Notice that the explicit recursive filter for exact filtering system was previously derived only for maximal rank case (i.e.. rank of (c_{ij}) is n), see [Be] for a particular maximal rank case and [T-W-Y] for general maximal rank case. Our result is an important breakthrough for the last 20 years because our method works for most general filtering systems and we no longer need maximal rank condition to construct explicit recursive filter and we can allow arbitrary initial condition. The significance of our results are that our algorithm is universal for any Yau filtering system with arbitrary initial condition and that this estimation problem have been factored into two parts: (1) the off-line calculation of the Kolmogorov type equation which does not depend on the observations and (2) the on-line solution of a finite linear system of ordinary differential equations, which can be realized in real time.

(II) Although the concept of estimation algebra has proven to be an invaluable tool in the study of nonlinear filtering problems [Ma], until recently very little was known about

estimation algebras. Beginning in the late 1980s, however, the structure and classification of finite-dimensional exact estimation algebras were studied in detail [T-W-Y] [D-T-W-Y]. In [Wo], the concept of Ω was introduced, which is defined as the matrix whose (i,j) element is $\frac{\partial f_j}{\partial x_i} - \frac{\partial f_i}{\partial x_j}$, where f is the drift term of the state evolution equation. For the class of exact filtering systems, Ω is identically zero. More recently, Yau [Ya1], [Ya2] has studied filtering systems such that all entries of Ω are constants. He was able to classify all finite-dimensional estimation algebras of maximal rank in such filtering systems. Chiou-Yau [Ch-Ya] and Chen-Leung-Yau [C-L-Y1], [C-L-Ya2] have shown respectively that if the dimension of the state space is two or three or four, all entries of Ω are constants as long as the estimate algebra is of maximal rank and finite dimensional. Thus, finite-dimensional estimation algebra of maximal rank is completely classified if the dimension of state space is at most three. The novelty of their theorems is that there are no a priori assumptions on the drift term of the nonlinear filtering system.

Our approach for the complete classification of finite-dimensional estimation algebras of maximal rank consists of two steps. The first step is to prove that for such an estimation algebra, all the entries in the Ω-matrix are degree one polynomials. The second step is to prove that in fact all the entries in Ω are constants. Then we can apply the result of Yau [Ya] to give a complete classification of finite-dimensional estimation algebras of maximal rank. Recently we have completed the first step (cf. [Ch-Ya1]). The following theorem is due to Hu and Yau [Hu-Ya2].

Theorem 3. If E is a finite-dimensional estimation algebra of maximal rank, then all the entries $\omega_{ij} = \frac{\partial f_j}{\partial x_i} - \frac{\partial f_i}{\partial x_j}$ of Ω are degree one polynomials. Let k be the maximal rank of quadratic forms in E. Then ω_{ij} are constants for $1 \le i,\, j \le k$ and $k+1 \le i,\, j \le n$; ω_{ij} are degree one polynomials in $x_1, \dots, x_k$ for $1 \le i \le k$ or $1 \le j \le k$.

Let n be the dimension of the state space. In case $n = 3$, there are three unknowns: ω_{12}, ω_{13}, and ω_{23}. It is easy to see that they are all degree two polynomials in view of Ocone's theorem. In [Ya-Le], Leung and Yau showed that the coefficients of the quadratic parts of ω_{12}, ω_{13}, and ω_{23} have to satisfy 90 quadratic equations. It was also shown in that paper that this system of 90 quadratic equations has only a trivial solution. Later Chen, Leung, and Yau proved that Ω is a matrix of constants [C-L-Y1]. The novelty of our main theorem is that it holds for arbitrary n. Thus, it is the fundamental step in the classification of finite-dimensional estimation algebras of maximal rank.

References

[Ba] Baras, J.S., Group invariance methods in nonlinear filtering of diffusion processes, in *Stochastic systems: The Mathematics of Filtering and Identification and Applications,* M. Hazewinkel and J.C. Willems, eds., Reidel, Dordrecht, 1981.

[Be] Benés, V., Exact finite dimensional filters for certain diffusions with nonlinear drift, *Stochastics* **5** (1981), 65-92.

[Be-Ka] Benés, V.E., and Karatzas, I., Estimation and Control for linear partially observable systems with non-Gaussian initial distribution, *Stochastic Processes and Their Application*

14 (1983), 233-248.

[Br] Brockett, R.W., Nonlinear systems and nonlinear estimation theory, in *The Mathematics of Filtering and Identification and Application,* M. Hazewinkel and J.S. Willems, eds., Reidel, Dordrecht, 1981.

[Br-Cl] Brockett, R.W., and Clark, J.M.C., The geometry of the conditional density functions, in *Analysis and Optimization of Stochastic Systems,* O.L.R. Jacobs, et al., eds., Academic Press, New York, 1980, 299-309.

[Ch] Chen, J., On Ubiquity of Yau Filters, Proceedings of the American Control Conference, Baltimore, Maryland, June (1994), 252-254.

[Ch-Ya] Chiou, W.L., and Yau, S.S.-T., Finite dimensional filters with nonlinear drift II: Brockett's problem of classification of finite dimensional estimation algebras, *SIAM J. Control and Optimization* **32**, no. 1 (1994), 297-310.

[Ch-Ya1] Chen, J. and Yau, S.S.-T., Finite dimensional filters with nonlinear drift VI: Linear structure of Ω matrix, Mathematics of Control, Signals and Systems, **9** (1996), 370-385.

[Ch-Ya2] Chen, J. and Yau, S.S.-T., Finite dimensional filters with nonlinear drift VII: Mitter conjecture and structure of η (with Jie Chen), *SIAM J. Control and Optimization,* **35**, no. 4, July (1997), 1116-1131.

[C-L-Y1] Chen, J., Leung, C.W., and Yau, S.S.-T., Finite dimensional filters with nonlinear drift IV: Classification of finite dimensional estimation algebras of maximal rank with state space dimension three, *SIAM J. Control and Optimization* **34**, no. 1, (1996), 179-198.

[C-L-Y2] Chen, J., Leung, C.W., and Yau, S.S.-T., Finite dimensional filters with nonlinear drift VIII: Classification of finite dimensional estimation algebra of maximal rank with state space dimension 4, *SIAM J. Control and Optimization,* **35**, no. 4, July (1997), 1132-1141.

[D-T-W-Y] Dong, R.T., Tam, W.S., Wong, W.S., and Yau, S.S.-T., Structure and classification theorems of finite dimensional exact estimation algebras, *SIAM J. Control and Optimization* **29** (1991)866-877.

[D-W-Y] Dong, R.T., Wong, W.S., and Yau, S.S.-T., Filtering system with finite dimensional estimation algebras, *IEEE Automatic Control,* **42** (11), (1997), 1601-1606.

[Ha] Hazewinkel, M., Lectures on linear and nonlinear filtering, in Analysis and Estimation of Stochastic Mechanical Systems, CISM Courses and Lectures 303, W. Shiehlen and W. Wedig, eds., Springer, Vienna 1988, 103-135.

[Ha-Pa] Haussmann, U.G., and Pardouxx, E., A conditionally almost linear filtering problem with non-Gaussian initial condition, *Stochastics* **23** (1988), 241-275.

[Hu-Ya1] Hu, G.Q. and Yau, S.S.-T., Explicit solution of DMZ equation, Proceedings of the 36th IEEE Conference on Decision and Control, San Diego, California, Dec. (1997), 4455-4459.

[Hu-Ya2] Hu, G.Q. and Yau, S.S.-T., Finite dimensional filters with nonlinear drift XII: Linear and constant structure of Ω, (preprint).

[Hu-Ya3] Hu, G.Q. and Yau, S.S.-T., Finite dimensional filters with nonlinear drift XIII: Classification of finite dimensional estimation algebras of maximal rank with state space dimension less than or equal to five, (preprint).

[L-Y-Y] Liang, Z.G., Yau, S.-T., and Yau, S.S.-T., Finite dimensional filters with nonlinear drift V: Solution to Kolmogorov equation arising from linear filtering with non-Gaussian initial condition, *IEEE Transactions on Aerospace and Electronic Systems,* **33**, no. 4, October (1997), 1295-1308.

[Ma] Marcus, S.I., Algebraic and geometric methods in nonlinear filtering, *SIAM J. Control and Optimization* **22** (1984), 817-844.

[Ma 1] Makowski, A., Filtering formulae for partially observed linear systems with non-Gaussian initial conditions, *Stochastics* **16** (1986), 1-24.

[Mi 1] Mitter, S.K., On the analogy between mathematical problems of nonlinear filtering and quantum physics, *Ricerche di automatica* **10**, no. 2 (1979), 163-216.

[Mi 2] Mitter, S.K., Lectures on nonlinear filtering and stochastic control, in *Nonlinear Filtering and Stochastic Control,* eds. S.K. Mitter and A. Moro, Springer Lecture Notes in Mathematics #972 (1983), 170-207.

[Oc] Ocone, D., Topics in nonlinear filtering theory, Ph.D. Dissertation, Department of Mathematics, Massachusetts Institute of Technology, 1980.

[Ra-Ya] Rasoulian, A. and Yau, S.S.-T., Finite dimensional filters with nonlinear drift IX: Construction of finite dimensional estimation algebra of non-maximal rank, Systems Control Letters, **30** (1997), 109-118.

[T-W-Y] Tam, L.F., Wong, W.S., and Yau, S.S.-T., On a necessary and sufficient condition for finite dimensionality of estimation algebras, *SIAM J. Control and Optimization,* **28**, no. 1 (1990), 173-185.

[Wo 1 Wong, W.S., On a new class of finite dimensional estimation algebras, *Systems and Control Letters* **9** (1987), 79-83.

[Wo 2] Wong, W.S., Theorems on the structure of finite dimensional estimation algebras, *Systems and Control Letters* **9** (1987), 117-124.

[Wo-Ya] Wong, W.S. and Yau, S.S.-T., The estimation algebra of nonlinear filtering systems. in Mathematical Control Theory,J. Baillieul and J.C.W. Willems, eds., Springer-Verlag, (1998), 33-65.

[Ya1] Yau, S.S.-T., Recent results on nonlinear filtering: New class of finite dimensional filters, Proceedings of the 29th Conference on Decision and Control at Honolulu, Hawaii, Dec. (1990), 231-233.

[Ya2] Yau, S.S.-T., Finite dimensional fitlers with nonlinear drifts I: A class of filters containing both Kalman filters and Benés filters, *J. Math. Systems, Estimation and Control* **4**, no. 2 (1994), 181-203.

[Ya-Le] Yau, S.S.-T. and Leung, C.-W., Recent result on classification of finite dimensional maximal rank estimation algebras with state space dimension 3, *Proceedings of the 31st Conference on Decision and Control*, Tucon, Arizona, Dec. (1992), 2247-2250.

[Ya-Ya 1] Yau, S.-T., and Yau, S.S.-T., Explicit solution to a Kolmogorov equation, *Applied Mathematics and Optimization*, an international journal (to appear).

[Ya-Ya 2] Yau, S.S.-T. and Yau, S.-T., New direct method for Kalman-Bucy filtering system with arbitrary initial condition, in *Proceedings of the 33rd Conference on Decision and Control*, Lake Buena Vista, Florida, Dec. (1994), 1221-1225.

[Ya-Ya3] Yau, S.-T., and Yau, S.S.-T., Finite dimensional fitlers with nonlinear drift XI: Explicit solutions of the generalized Kolmogorov equation in the Brockett Mitter program, *Advances in Mathematics,* **140** (1998), 156-189.

C. List of all Publications and Technical Reports

1. The wavelet application to Kolmogorov equation, (with Zhigang Liang) Proceeding of International Conference on Control and Information, (1995), 271-276.
2. A report on explicit formulas for exp(tA), (with Hon-wing Cheng) Proceeding of International Conference on Control and Information, (1995), 69-75.
3. Recent results on classification of 4-dimensional estimation algebras (with Amid Rasoulian) Proceeding of International Conference on Control and Information, (1995), 371-374.
4. Random wavelet transfrom, algebraic geometric coding and their applications in compression and de-noising of signals, (with Tomasz Bielecki; Man K. Kwong; Li M. Song) Proceeding of International Conference on Control and Information, (1995), 283-289.
5. Explicit construction of finite dimensional nonlinear filters with state space dimension 3, (with Jie Chen and Chi-Wah Leung) Proceedings of the 34th IEEE Conference on Decision and Control, New Orleans, Louisiana, Dec. 13-15, (1995). 4030-4034.
6. Construction of new finite dimensional nonlinear filters, (with Amid Rasoulian) Proceedings of the 34th IEEE Conference on Decision and Control, New Orleans, Louisiana, Dec. 13-15, (1995), 4002-4005.
7. Finite dimensional filters with nonlinear drift IV: classification of finite dimensional maximal rank estimation algebra with dimension of state space equal to 3, (with Jie Chen, Chi-Wah Leung), SIAM J. Control and Optimization, Vol. 34, No.1 (1996). 179-198.
8. Direct method without Riccati equation for Kalman-Bucy filtering system with arbitrary initial condition, (with Guo-Qing Hu), 13th World Congress IFAC. San Francisco, June 30-July 5, Volume H, (1996), 469-474.
9. Iconic indexing and maintenance of spatial relationships in image databases, (with Qing-Long Zhang and Shi-Kuo Chang) 12 pp. in ms. Conference on Multimedia Storage and Archiving Systems, SPIE's Photonics East 96 Symposium, Boston, MA, Nov. 18-22, 1996.
10. Explicit solution of a Kolmogorov equation (with S.T.Yau), Applied Mathematics and Optimization, An International Journal, 34, (1996), 231-266.
11. A unified approach to indexing images in image databases, (with Qing-Long Zhang and Shi-Kuo Chang), Proceedings of First International Workshop on Image Databases and Multi-Media Search, Amsterdam, The Netherland, August 22-23, 1996,99-106.
12. The Structure of Ω-matrix in nonlinear filters (with Amid Rasoulian),Proceedings of the 35th Conference on Decision and Control, Kobe, Japan (1996).1083-1087.
13. Parallel ODE-solvers for Kalman-Bucy Filter with Arbitrary Initial Condition (with Ho-Wing Cheng), Proceedings of the 35th Conference on Decision and Control, Kobe, Japan (1996), 4138-4145.

need reprint

14. New direct method for Yau filtering system with arbitrary initial conditions (with Guo-Qing Hu), Proceedings of the 35th Conference on Decision and Control, Kobe, Japan (1996),2539-2544.
15. A unified approach to iconic indexing, retrieval and maintenance of spatial relationships in image databases, (with Qing-Long Zhang and Shi-Kuo Chang), Journal of Visual

Communication and Image Representation Vol. 7, No. 4 (Special Issue), Dec. (1996), 307-324.

16. Finite dimensional filters with nonlinear drift VI: Linear structure of Ω matrix,(with Jie Chen), Mathematics of Control, Signals and Systems, 9(1996), 370-385.
17. Finite dimensional filters with nonlinear drift III: Duncan-Mortenson-Zakai equation with arbitrary initial condition for Kalman-Bucy filtering system and Benes filtering system (with Shing Tung Yau), IEEE Transactions on Aerospace and Electronic Systems, Vol. 33, No. 4. Oct. (1997), 1277-1294.
18. More explicit formulas for the matrix exponential (with Hon Wing Cheng),Linear Algebra and Application, vol 262, (1997), 131-163.
19. Finite dimensional filter with nonlinear drift V: Solution to Kolmogorov equation arising from linear filtering with non Gaussian initial condition (with Zhigang Liang, and Shing -Tung Yau), IEEE Transactions on Aerospace and Electronic Systems, Vol. 33, No. 4, Oct. (1997), 1295-1308.
20. Finite dimensional filters with nonlinear drift VII: Mitter conjecture and structure of η (with Jie Chen). SIAM J. Control and Optimization, Vol. 35, No. 4, July (1997), 1116-1131.
21. Finite dimensional filters with nonlinear drift VIII: Classification of finite dimensional estimation algebra of maximal rank with state space dimension 4 (with Jie Chen and Chi-Wah Leung). SIAM J. Control and Optimization,Vol. 35,No. 4, July (1997), 1132-1141.
22. Finite dimensional filters with non-linear drift IX: construction of finite dimensional estimation algebra of non-maximal rank (with Amid Rasoulion), Systems Control Letters, 30(1997), 109-118.
23. Contribution to Munuera's Problem on the Main Conjecture of Geometric Hyperelliptic MDS Codes (with Hao Chen), IEEE Trans. Inform. Theory. Vol. 43, No. 4, (1997), 1349-1354.
24. A new iconic indexing for 2D and 3D scenes (with Qing-Long Zhang), Proceeding of the Second Chinese World Congress on Intelligent Control and Intelligent Automation (CWCICIA 97), Xian, June 23-27, 1997, 1667-1672.
25. Explicit solution of DMZ equation (with Guo-Qing Hu), Proceedings of the 36th IEEE Conference on Decision and Control, San Diego, California, Dec.(1997), 4455-4459.
26. Explicit construction of finite-dimensional nonlinear filters with state space dimension 4 (with Chi-Wah Leung, Jie Chen),Proceedings of the 36th IEEE Conference on Decision and Control, San Diego, California, Dec.(1997), 4462-4466.
27. Filtering system with finite dimensional estimation algebras (with Rui-Tao Dong and Wing-Shing Wong . IEEE Automatic Control, 42(11), (1997), 1601-1606.
28. An Experimental Result in Image Indexing using GEP-2D Strings (with Qing-Long Zhang and Shi-Kuo Chang), Image Database and Multimedia Search (eds. A. Smeulders and R. Jain), World Scientific Publ. Co., 1997, 127-146.
29. Addendum to "Finite -Dimensional Filters with Nonolinear Drift": A Response to F. Daum's Comments. (with Shing Tung Yau), IEEE Transactions on Aerospace and Electronic Systems, 34. No.2 (1998), 57-58.
30. The estimation algebra of nonlinear filtering systems (with W.S. Wong), Mathematical

Control Theory, Special Volume Dedicated to 60th Birthday of Brockett, J. Baillieul and J.C. Willems eds., Springer Verlag, (1998), 33-65.

31. Finite dimensional filters with nonlinear drift X: Explicit Solution of DMZ Equation (with Guo-Qing Hu), 15 pp. in ms (to appear)IEEE Transactions on Automatic Control.
32. Classification of Four-Dimensional Estimation Algebras (with Amid Rasoulian), 13 pp. in ms., (to appear) IEEE Transactions on Automatic Control.
33. Wavelet representations of general signals, (with T. Bielecki, J. Chen and E. Lin), Nonlinear Analysis, Vol. 35, no. 1, (1999), 125-141.
34. Finite Dimensional Filters with Nonlinear Drift XI: Explicit Solution of the Generalized Kolmogorov Equation in the Brockett- Mitter Program (with Shing-Tung Yau), Advances in Mathematics, 140, (1998), 156-189.
35. Finite dimensional filters with nonlinear drift XII: Linear and constant structure of Ω (with GuoQing Hu), 16 pp. in ms, submitted for publication.
36. Finite dimensional filters with nonlinear drift XIII: Classification of finite-dimensional estimation algebras of maximal rank with state space dimension less than or equal to five(with GuoQing Hu), 33 pp. in ms, submitted for publication.
37. Recent results on classification of finite dimensional maximal rank estimation algebras with state space dimension up to 6 (with Guo-Qing Hu), Proceedings of the 37th IEEE Conference on Decision and Control, Dec. 16-18, Tampa, Fl, 1998, 1311-1312.
38. Recent advance in computing Kolmogorov equation arising from nonlinear filtering (with Xi Wu and Ke-Pao Lin), Proceedings of the 37th IEEE Conference on Decision and Control, Dec. 16-18, Tampa, Fl, 1998, 2904-2905.
39. Finite dimensional filters with nonlinear drift XV: New direct method for construction of universal finite-dimensional filter (with Guo-Qing Hu), 11 pp. in ms, submitted for publication.
40. Existence and Uniqueness and Decay Estimates for the Time Dependent Parabolic Equation with Application to Duncan-Mortensen-Zakai equation (with S.T.Yau), 35 pp. in ms, (to appear) Asian Journal of Mathematics.

D. List of all participating scientific personnel

Senior Personnel:

Professor Stephen S.T. Yau	University of Illinois at Chicago
Professor S.T. Yau	Harvard University
Professor W.-S. Wong	Chinese University of Hong Kong
Professor T. Bielecki	Northeastern Illinois University

Junior Personnel:

Dr. J. Chen	University of Illinois at Chicago
Dr. X.Z. Yang	University of Illinois at Chicago
Professor C.W. Leung	Open University of Hong Kong
Professor Amid Rasoulian	Tehran University
Dr. G.Q. Hu	United Chamber Inc.

Graduate Students

Ph.D. Students:

Amid Rasoulian, Ph.D.
Hon Wing Cheng, Ph.D.
Qing-Long Zhang, Ph.D.
Guo-Qing Hu, Ph.D.
Zhi-Gang Liang
Xuejun Wang
Yi Wang
Ke-Pao Lin
Ebenezer Armah

Master Students:

Ebenezer Armah, M.A.
Xi Wu, M.A.
Xiaobo Wang
Yi Chen, M.A.
Guoqing Hu, M.A.
Xuejun Wang, M.A.
Jin Chen, M.A.
Xiande Xie, M.A.
Yan Dai, M.A.
Chen Zhang, M.A.
Weifang Zhou, M.A.
Lan Zuo, M.A.
Yan Chen, M.A.
Guoping Jiang, M.A.
Dajun Liang, M.A.

Report of Inventions

The best algorithms for Yau filtering system with arbitrary initial conditions were found. These algorithms are of practical importance.